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Active control of laminar stability by feedback control strategy

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Abstract

A new active way to control laminar stability through feedback control strategy is presented in this paper. By applying the expansion method, the small perturbation parameter, which is related to uniform boundary heat flux, is well separated out from the problem we studied. As a result, the heat flux independent coefficients, which reflect the temperature dependent viscosity effects, can be obtained. It is shown that this feedback laminar flow control strategy can considerably dominate the development of fluctuations provided that the phase of heating or cooling and the control position are properly chosen.

Keywords: Control; Heat transfer; Instability

1. Introduction

The problem of stability and transition becomes more and more important with the development of gas-turbine-engine, low-Reynolds-number vehicles, submarines, airplanes, space ships, etc. To delay/advance transition, to suppress/enhance turbulence or to prevent/provoke separation by various techniques, which are called flow control, always results in drag reduction, lift enhancement, mixing enhancement and flow-induced noise suppression. Thus, Gad-el-Hak [1] pointed out that flow control is perhaps more hotly pursued by scientists and engineers than any other areas of fluid mechanics. Many flow control strategies such as heating or cooling, suction or injection, flexible surface, compliant coating and large-eddy breaking have been recently developed, see the review papers of Morkovin and Reshotko [2] and Gad-el-Hak [1] for details.

The pioneering work of Wazzan et al. [3] showed that for a heated flat plate boundary layer in water, the critical Reynolds number can be increased from 520 to nearly 16000 (based on displacement thickness). Thus, there is a considerable potential for flow control with heating or cooling. But uniform heating or cooling are always accompanied with relatively large energy cost; therefore, optimal steady heating or cooling is recently studied [4,5].

A more effective active control technique called wave cancellation by localized periodical surface heating strips was reported by Liepmann et al. [6,7].

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Nomenclature

a	general quantity
$\hat{a}$	shape function of a
c_i	amplification rate
c_p	specific heat at constant pressure
c_r	phase velocity
D	derivative with respect to y
H	half channel height
k	thermal conductivity
K_d	controller gain
K_μ	nondimensional viscosity gradient, Eq. (6)
p	pressure
Pr	Prandtl number $\frac{\mu_R^* c_p^*}{k^*}$
q_w	wall heat flux
Re	Reynolds number $\frac{\rho^* U_R^* H^*}{\mu_R^*}$
t	time
T	temperature
$\hat{T}$	temperature shape function
u	streamwise velocity
U_R	reference velocity
v	velocity normal to the wall
x, y	Cartesian coordinates

Greek symbols

α	wave length parameter
ϵ	perturbation parameter, Eq. (6)
μ	viscosity
ψ	stream function
ρ	density
θ	control phase angle

Superscripts

$*$	dimensional quantity
$-$	mean value
$\sim$	disturbance quantity
$\wedge$	complex quantity

Subscripts

c	critical or extreme point
i	imaginary part
r	real part
R	reference state
w	wall
0	zero order
μ	first order viscosity effect

In their boundary layer experiments in water, two heating elements are used: one for exciting T–S wave and the other for cancelling T–S wave. They found that the localized periodical surface heating can either reduce or enhance the overall level of flow field fluctuations. In addition, by measuring the upstream wall shear stress of the controlling surface, they were able to synthesize a signal to drive the cancellation disturbance at the controlling surface. A feedback control technique is established. Furthermore, they demonstrated that the energy cost for controlling flow field can be greatly reduced by applying this technique. This concept of active control was demonstrated later by the numerical simulation of Bayliss et al. [8] and the triple-deck asymptotic analysis of Maestrello and Ting [9]. More recently, Ladd and Hendricks [10] and Ladd [11] performed their experiment on a 9:1 fineness ratio ellipsoid in a water tunnel. Strip heaters were again used to creat and actively attenuate T–S wave. They applied digital filtering techniques to synthesize the attenuation signal. The filter was able to actively adapt the attenuation signal to change in amplitude and frequency of the artificially introduced instability wave with no loss of attenuation downstream. Kral and Fasel [12] developed a numerical model to control the spatial evolving T–S wave and its secondary instability on a flat plate boundary layer. Temperature perturbations are introduced locally along finite heater strips to directly attenuate the instability waves in the flow. Their results showed that the active control of the early stages of fundamental breakdown process is achieved with either two-dimensional or three-dimensional control inputs. Asymptotic studies of Herwig et al. [13,14] showed that the temperature disturbance can be passively generated by the interaction of T–S wave and mean temperature profile through the energy equation. On the other hand, the temperature disturbance can also influence the critical Reynolds number through the temperature dependent fluid properties such as viscosity. But this effect is only about 10% of that of the mean temperature [14].

A new feedback flow control technique is presented in this paper. This control strategy is an effective combination of the uniform heating or cooling of Wazzan et al. [3] with the localized periodical surface heating of Liepmann et al. [6,7]. The combination of these two techniques is very important and necessary because the control strategy of Liepmann et al. has a fatal shortcoming. In a natural transition procedure, the disturbances are a group of waves and the largest amplitude wave is the T–S wave. Although these primary waves are behaving linearly, a nonlinear interaction can cause a lower amplitude disturbance wave without the frequency of T–S wave to be only partially reduced and ultimately lead to transition. The new control technique can avoid this problem and largely reduce all wave disturbances through the uniform heating or cooling. This problem will be discussed again in Section 6.

Instead of measuring the upstream wall shear stress of the controlling surface as a feedback control strategy, a temperature sensor is used to measure the temperature perturbation in this paper. The feedback control is achieved by using the measured temperature fluctuation in the flow field as the input signal to control the downstream output periodical heat flux. This basically is to enhance the effects of the temperature disturbance on flow stability. By applying this mechanism, laminar flow stability is greatly enhanced or reduced.

The analysing method is the asymptotic approach of Herwig and Schäfer [13]. As a advantage of this approach, it provides results that hold for all Newtonian fluids and different heat transfer rates.

The plane Poiseuille flow with constant wall heat flux is chosen as our analysis example, see Fig. 1. Downstream of an adjustment zone, the flow will reach a fully developed state. Our feedback control will refer to this part of flow field, see Herwig and You [14] for details of the flow. Without losing generality, we can only study the case with the temperature dependent viscosity, which is a good approximation for water. All other variable properties can be treated likewise, see for example [15].

2. Basic equations

In the stability theory, all quantities are decomposed into a mean value $\bar{a}$, and a superimposed disturbance $\tilde{a}$ ($\tilde{\psi}$, stream function; $\tilde{T}$, temperature fluctuation). The form of disturbance part can be assumed as:

$$\tilde{a} = \hat{a}(y)\exp\left[i\alpha(x - \hat{c}t)\right] + c.c. \tag{1}$$

Then, from the Navier–Stokes equations (for temperature dependent viscosity) and thermal energy equation, the linear differential equations for $\hat{\psi}(y)$ and $\hat{T}(y)$ are deduced with $D = \frac{\partial}{\partial y}$:

$$\begin{aligned}&\left[(\bar{u} - \hat{c})(D^2 - \alpha^2) - D^2\bar{u}\right]\hat{\psi} \\ &\quad = -\frac{i}{\alpha Re}\left[\bar{\mu}(D^2 - \alpha^2)^2\right]\hat{\psi} - \frac{i}{\alpha Re}\left[\left(2D\bar{\mu}D\right.\right. \\ &\qquad \left.\left. + 2i\alpha\frac{\partial\bar{\mu}}{\partial x}\right)(D^2 - \alpha^2) + D^2\bar{\mu}\left(D^2 + \alpha^2\right)\right]\hat{\psi} \\ &\qquad - \frac{i}{\alpha Re}\left[D^3\bar{u} + \alpha^2 D\bar{u} + 2D^2\bar{u}D + D\bar{u}D^2\right]\hat{\mu},\end{aligned} \tag{2}$$

$$\begin{aligned}&\left[(\bar{u} - \hat{c}) + \frac{i}{\alpha Re\, Pr}(D^2 - \alpha^2)\right]\hat{T} \\ &\quad = \left(D\bar{T} + \frac{i}{\alpha}\frac{\partial\bar{T}}{\partial x}D\right)\hat{\psi}.\end{aligned} \tag{3}$$

Here, $Re = \frac{\rho^* U_R^* H^*}{\mu_R^*}$ and $Pr = \frac{\mu_R^* C_p^*}{k^*}$ are Reynolds number and Prandtl number, respectively. $\bar{u}$ and $\bar{\mu}$ are temperature dependent mean flow velocity and viscosity.

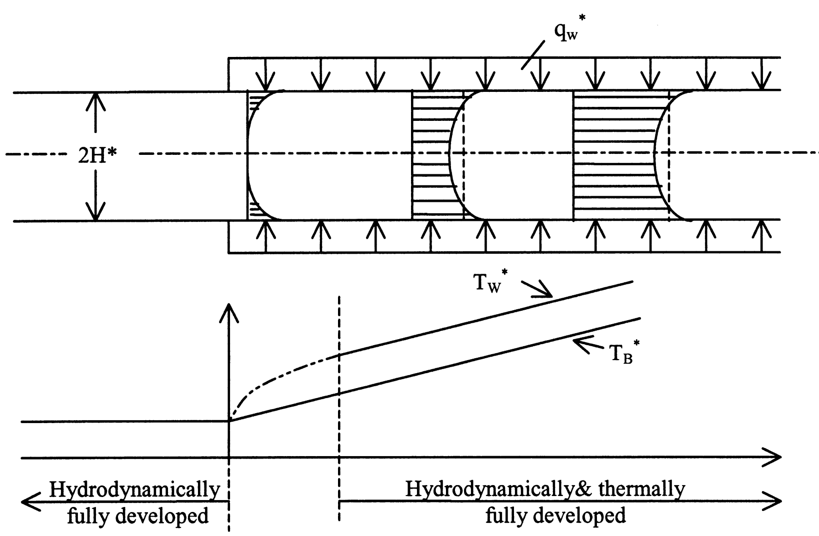

Fig. 1. Development of the temperature profile.

The associated boundary conditions for velocity disturbance are:

$$y = \pm 1: \quad \hat{\psi} = \frac{\partial \hat{\psi}}{\partial y} = 0. \tag{4}$$

The boundary condition for temperature disturbance $\hat{T}$ forms the basis of our feedback control strategy and will be defined in Section 5.

All the above equations are nondimensionalized with respect to a reference state R which may be chosen at any position x_R^* in the fully developed region. The reference velocity U_R^* is the maximum mean flow velocity with constant viscosity. The reference temperature T_R^* is the bulk temperature at x_R^*, i.e. $\bar{T}_B^*(x_R^*)$. The nondimensional temperature of the mean flow is $\bar{T} = \frac{\bar{T}^* - T_R^*}{\Delta T_R^*}$, and that of the fluctuation is $\tilde{T} = \frac{\tilde{T}^*}{\Delta T_R^*}$ with $\Delta T_R^* = \frac{\bar{q}_w^* H^*}{k^*}$, see Herwig and You [14] for details.

3. Property expansion method

The basic idea behind the property expansion method is to combine the Taylor series expansion of μ^* (or all properties in the general case) with respect to temperature with a regular perturbation procedure of the whole problem. From the Taylor series expansion of μ^*, which in nondimensional form reads:

$$\mu = \frac{\mu^*}{\mu_R^*} = 1 + \epsilon K_\mu T + O(\epsilon^2) \tag{5}$$

with

$$K_\mu = \left(\frac{d\mu^*}{dT^*} \frac{T^*}{\mu^*} \right)_R, \qquad \epsilon = \frac{\bar{q}_w^* H^*}{k^* T_R^*}. \tag{6}$$

a small quantity ϵ can be extracted which may serve as a perturbation parameter of the whole problem. Truncating the Taylor series after the linear term, results in a linear perturbation theory with respect to ϵ. Extension to higher orders (ϵ^2, ϵ^3, $\cdots$) is straightforward, see You and Herwig [16] for details, but it is not in the scope of this study. The parameter K_μ is a property of the fluid (for example: water at $T_R^* = 293$ K, $K_\mu = -7.134$).

According to Eq. (5), we expand:

$$\{\bar{u}, \bar{T}, \bar{\mu}\}^{\mathrm{T}} = \{\bar{u}_0, \bar{T}_0, 1\}^{\mathrm{T}} + \epsilon K_\mu \{\bar{u}_\mu, \bar{T}_\mu, \bar{T}_0\}^{\mathrm{T}} + O(\epsilon^2), \tag{7}$$

$$\left\{\hat{\psi}, \hat{T}, \hat{\mu}, \hat{c}\right\}^{\mathrm{T}} = \left\{\hat{\psi}_0, \hat{T}_0, 0, \hat{c}_0\right\}^{\mathrm{T}} + \epsilon K_\mu \left\{\hat{\psi}_\mu, \hat{T}_\mu, \hat{T}_0, \hat{c}_\mu\right\}^{\mathrm{T}} + O(\epsilon^2). \tag{8}$$

In the above expansions, the quantities with index 0 describe the constant property behavior and those with index μ reflect the influence of viscosity deviations due to its temperature dependence.

The fully developed mean flow field solution, i.e. $\bar{u}_0$, $\bar{T}_0$, $\bar{u}_\mu$ and $\bar{T}_\mu$ in Eq. (7) can be given analytically as:

$$\bar{u}_0 = \left(1 - y^2\right),$$

$$\bar{T}_0 = \frac{3}{2}\left(-\frac{1}{12}y^4 + \frac{1}{2}y^2 - \frac{13}{140} \right) + \frac{3x}{2Re\,Pr},$$

$$\bar{u}_\mu = -\frac{1}{24}y^6 + \frac{3}{8}y^4 - \frac{111}{280}y^2 + \frac{53}{840},$$

$$\bar{T}_\mu = \frac{1}{2}\left(-\frac{1}{448}y^8 + \frac{3}{80}y^6 - \frac{111}{1120}y^4 + \frac{53}{560}y^2 + \frac{16{,}917}{2{,}587{,}200} \right). \tag{9}$$

The reference point R is taken at $x = 0$ in our calculation.

4. Linear stability equations

If we now insert Eqs. (7) and (8) into Eqs. (2) and (3), we end up with the following zeroth and first order equations with respect to ϵ. But, before we do this, we split the first order function $\hat{\psi}_\mu$ and $\hat{c}_\mu$, which reflect the change of velocity disturbance and wave phase velocity, respectively due to temperature dependent viscosity, into two parts, i.e. we write:

$$\hat{\psi}_\mu(T) = \hat{\psi}_{\mu 1}(\bar{T}) + \hat{\psi}_{\mu 2}(\hat{T}), \tag{10}$$

$$\hat{c}_\mu(T) = \hat{c}_{\mu 1}(\bar{T}) + \hat{c}_{\mu 2}(\hat{T}). \tag{11}$$

Thus, the influence of the temperature disturbance $\hat{T}$ is well separated from that of the mean temperature $\bar{T}$. This separation is possible due to the linearity of Eq. (2). Then we have:

zeroth order

$$L\hat{\psi}_0 = 0, \tag{12}$$

$$L_T \hat{T}_0 = \left(D\bar{T}_0 + \frac{3i}{2\alpha Re\,Pr} D \right) \hat{\psi}_0, \tag{13}$$

first order

$$L\hat{\psi}_{\mu 1} = -\Big[(\bar{u}_\mu - \hat{c}_{\mu 1})(D^2-\alpha^2)\hat{\psi}_0 - D^2\bar{u}_\mu - \frac{3}{Re^2\,Pr}(D^2-\alpha^2)\Big]\hat{\psi}_0 - \frac{i}{\alpha Re}\Big[\bar{T}_0(D^2-\alpha^2)^2 + 2D\bar{T}_0(D^2-\alpha^2)D + D^2\bar{T}_0(D^2+\alpha^2)\Big]\hat{\psi}_0, \quad (14)$$

$$L\hat{\psi}_{\mu 2} = \hat{c}_{\mu 2}(D^2-\alpha^2)\hat{\psi}_0 - \frac{i}{\alpha Re}\big[D^3\bar{u}_0 + 2D^2\bar{u}_0 D + D\bar{u}_0(D^2+\alpha^2)\big]\hat{T}_0. \quad (15)$$

Here

$$L = (\bar{u}_0 - \hat{c}_0)(D^2-\alpha^2) - D^2\bar{u}_0 + \frac{i}{\alpha Re}(D^2-\alpha^2)^2, \quad (16)$$

$$L_T = (\bar{u}_0 - \hat{c}_0) + \frac{i}{\alpha Re\,Pr}(D^2-\alpha^2). \quad (17)$$

The associated boundary conditions are:

$$y = \pm 1: \quad \hat{\psi}_0 = D\hat{\psi}_0 = \hat{\psi}_{\mu 1} = D\hat{\psi}_{\mu 1} = \hat{\psi}_{\mu 2} = D\hat{\psi}_{\mu 2} = 0. \quad (18)$$

The boundary condition for the temperature disturbance $\hat{T}_0$ will be defined in next section.

Eq. (12) is the well known Orr–Sommerfeld equation in standard form, Eqs. (14) and (15) are its first order extension to account for temperature dependent viscosity by the perturbation ansatz (7), (8), (10) and (11).

5. The feedback control strategy

The remaining boundary condition for the temperature disturbance forms the basis of the present feedback control. Before we look for the suitable temperature disturbance boundary condition, let's first consider the important role of the temperature fluctuation in our feedback control strategy. The study of Herwig and You [14] showed that the temperature disturbance can be passively generated by the interaction of T–S wave and mean temperature profile through the energy equation. They discussed in detail how a temperature disturbance is evolved. They found that the final shape of the temperature disturbance can be reached in a short time (about 1.6 wave period for the case of $Re = 5800$, $Pr = 7$ and $\alpha = 1.02$). We assume that the final shape of the temperature perturbation is reached in this study before we begin to perform our feedback control. On the other hand, they also showed that the temperature disturbance can affect the critical Reynolds number through the temperature dependent fluid properties such as viscosity, and they further found that this effect is only about 10% of that of the mean temperature. The basic idea behind this feedback control is to enhance the temperature disturbance influence on flow stability by the periodical heating. Then our goal to control transition by few energy input can be realized.

We suppose to use a number of temperature sensors to measure the fluid temperature at some fixed plane ($y_c =$ constant). Then, from the difference between the measured temperatures and the known mean temperatures, the amplitude of temperature disturbance $\hat{T}_0(y_c)$ can be determined (for temporal modes, temperature sensors are placed along a time period and for spatial modes, temperature sensors are placed along a space period. Experiments are always corresponding to spatial modes). We adjust the input heat flux disturbance (input heat flux minus the uniform heat flux) at flow field boundaries (two walls for our example) to be a function of the temperature fluctuation $\hat{T}_0(y_c)$ as $f(\hat{T}_0(y_c))$. f is called controller function. Now our feedback control loop is established.

It will be convenient if we can use the temperature disturbance on the wall as our feedback control input signal. Unfortunately, the temperature disturbance on the control wall is too small (near zero) to be used as the upstream control input signal. The control position y_c is chosen here. It corresponds to the point where the maximum value of the module of the temperature disturbance $\hat{T}_0(y)$ is realized before performing the feedback control. The large value of $\hat{T}_0(y_c)$ can be easily measured, and the feedback control is efficiently achieved.

Generally, the controller function is a nonlinear function. For a simplified case, we choose a linear function as the controller function in this paper. Other control functions can be treated in the same way. Then the boundary condition for temperature disturbance can be one of the following three cases:

$$D\hat{T}_0(\pm 1) = K_d e^{i\theta}\hat{T}_0(y_c), \quad (19)$$

$$D\hat{T}_0(1) = K_d e^{i\theta}\hat{T}_0(y_c), \qquad D\hat{T}_0(-1) = 0, \quad (20)$$

$$D\hat{T}_0(\pm 1) = \pm K_d e^{i\theta}\hat{T}_0(y_c). \quad (21)$$

Here K_d is controller gain, θ is control phase angle which is the difference between the phase of controlled input heat flux disturbance at upper wall ($y = 1$) and that of the temperature disturbance $\hat{T}_0(y_c)$.

The boundary condition (19) corresponds to generate a symmetric temperature disturbance which has no contribution to the first order coefficient $\hat{c}_{\mu 2}(\hat{T}_0)$. Thus, the feedback control effects are minimized. On the other side, the boundary condition (21) enhances the nonsymmetrical temperature disturbance which has the largest contribution to the first order coefficient $\hat{c}_{\mu 2}(\hat{T}_0)$. Then the maximum feedback control effects are realized. The boundary condition (20) means only the periodical heat flux of one wall (upper) is controlled. In this case, its effects on flow control are in the range of the above two extreme cases. Thus, the boundary condition (21) is adopted in all calculations.

6. Numerical results and discussion

The numerical method, which is used to solve Eqs. (12)–(15) and their boundary conditions (18) and (21), is Chebyshev tau method. All $\hat{\psi}_j$ and $\hat{T}_j$ are expanded in Chebyshev polynomials. For our case, 36 Chebyshev polynomials are appropriate. Our normalized condition is $\max(D\hat{\psi}_0) = 1$. All results are corresponding to water with $T_R^* = 293$, $K_\mu = -7.134$ and $Pr = 7$.

Fig. 2 shows the dependence of critical wave number α_c, minimum unstable Reynolds number Re_c, optimal control position y_c and control phase angle θ_c on the controller gain K_d with $\epsilon = 0.01$ (heating water). y_c and θ_c are the control values which correspond to the maximum feedback control. It is shown that as K_d increases, α_c decreases and Re_c and θ_c increases, but y_c is nearly constant.

It is well known that for the plane Poiseuille flow without heat transfer effects, the minimum unstable Reynolds number Re_c is about 5772 which is corresponding to the wave number $\alpha_c = 1.02$. If constant heat flux is input on two walls, the uniform heating will increase the minimum unstable Reynolds number

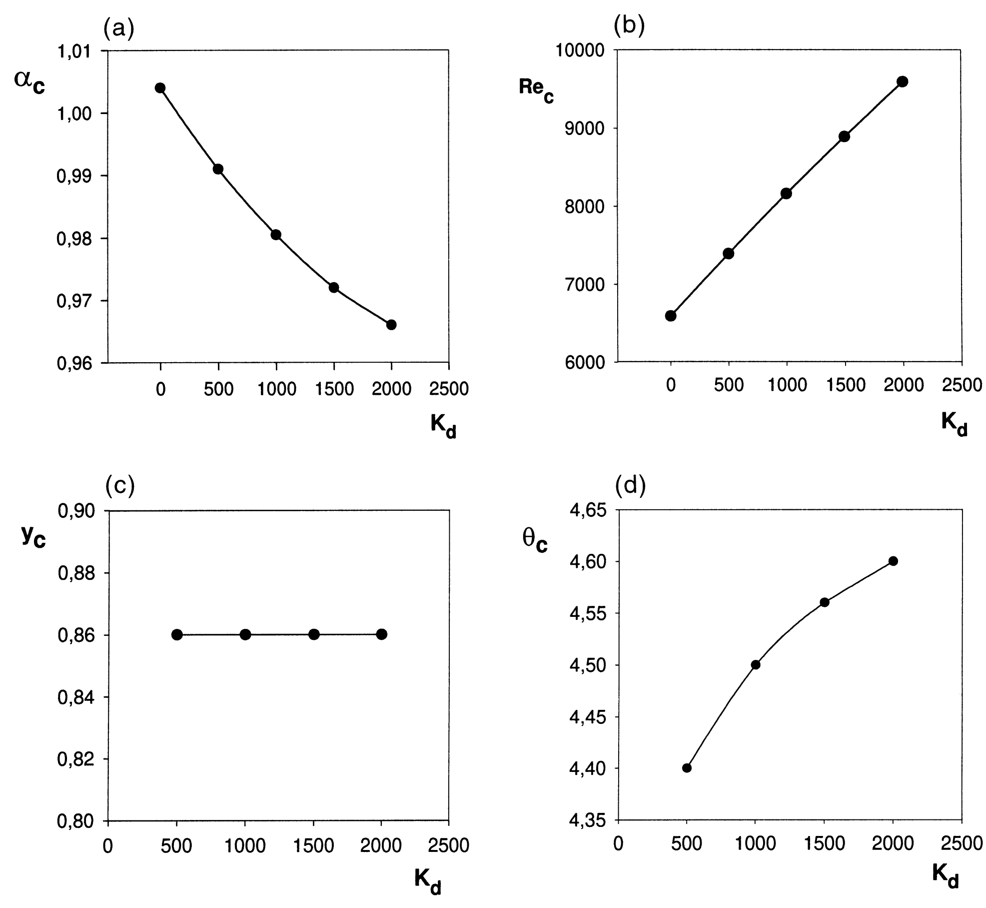

Fig. 2. The relation of critical wave number α_c, minimum unstable Reynolds number Re_c, optimal control position y_c and control phase angle θ_c with the controller gain K_d. $\epsilon = 0.01$ and $Pr = 7$. (a) α_c depends on K_d, (b) Re_c depends on K_d, (c) y_c depends on K_d, (d) θ_c depends on K_d.

Re_c for water. For example, the minimum unstable Reynolds number Re_c will increase to about 6593 as the heat transfer rate is $\epsilon = 0.01$. This case corresponds to our feedback control with zero controller gain.

The input heat flux can be separated as $q_w = \bar{q}_w + \epsilon\hat{q}_w$. For spatial modes, $\bar{q}_w$ is the average of q_w on space and for temporal modes, $\bar{q}_w$ is the average of q_w on time like it in this paper. The $\bar{q}_w$ is the uniform heat flux on the wall and is related to the small expansion parameter ϵ. For example, the minimum unstable Reynolds number is increased by about 821 when $\epsilon = (\bar{q}_w^* H^*)/(k_R^* T_R^*) = 0.01$. For a channel width of $2H^* = 0.1$ m, $T_R^* = 293$ K and $k_R^* = 0.6$ W/(m K) (water), this corresponds to a heat flux $\bar{q}_w^* = 35.16$ W/m^2. According to Eq. (9) for $\bar{T}_0$, this heat flux would cause a zeroth order temperature difference $\bar{T}_{0w}^* - \bar{T}_{0c}^* = 1.83$ K between the wall and the centerline. This may demonstrate that even for a small energy cost, there is an appreciable effect on the stability behavior of this flow. A further improvement would be using optimal steady heating or cooling instead of uniform heating or cooling. This may lead to further reduction of energy cost. It will be the next step of our study.

The $\hat{q}_w = k_R K_d e^{i\theta} \hat{T}_0(y_c)$ is related to our feedback control strategy. Fig. 2(b) shows the dependence of Re_c on K_d. For the case $K_d = 2000$, the Re_c is about 9592. How large is the input amplitude of heat flux disturbance at this time? We recall the above example again. For $\alpha_c = 0.963$, $\epsilon\hat{u}_0 = 1\%$ and $y_c = 0.86$, the $\epsilon|\hat{T}_0(y_c)| = 0.0322$ and the amplitude of input periodical heat flux $\epsilon|\hat{q}_w^*| = \epsilon k_R^* \frac{\Delta T_R^*}{H^*} K_d |\hat{T}_0(y_c)| = 2264$ W/m^2. The net input of heat flux disturbance $\epsilon\hat{q}_w^*$ over a period of time for our temporal modes is very small (the average of $\hat{q}_w^*$ is zero in theory) due to some parts of flow field are heated and the other parts are cooled. For this case, the feedback control can keep laminar flow stable until $Re_c = 9592$. This shows how powerful this feedback control is!

The Liepmann feedback control works well when there appears only one dominated frequency disturbance (T–S wave) in their experiment [6,7] (two heating elements were used. one was for exciting T–S wave and the other was for cancelling the induced T–S wave). But this case is quite different from a natural transition where besides the T–S wave, there are always waves with a wide range of other frequency fluctuations. Some of them perhaps have a less small amplitude comparing to that of T–S wave. Although these primary waves are behaving linearly, a nonlinear interaction can cause a lower amplitude disturbance wave without the frequency of T–S wave to be only partially reduced. This means the remaining wave disturbances in the natural transition are generally larger than those in the deliberately excited T–S wave case after the wave cancellation is performed.

Why does this phenomenon appear? If we recall the localized periodical surface heating control in water, the flow field with heating leads to a fuller mean velocity profile. This is favorable for the disturbances stabilized. On the other side, the remaining part of flow field with cooling causes a thin mean velocity profile, and sometimes it even leads to a mean velocity profile with a inflexion. This may destabilize the disturbances with different frequency during the cancellation of T–S wave. The above conclusion is proved again if we compare the Fig. 2(d) with Fig. 3(b) of Liepmann and Nosenchuck [7]. The Fig. 2(d) and Fig. 3(b) show the cancellation results of cancelling a deliberately excited T–S wave and a natural T–S wave, respectively. The amplitude of remaining disturbances in Fig. 3(b) are obviously larger than those in Fig. 2(d) and the larger remaining disturbances may lead to a laminar transition soon.

The new control strategy is an effective combination of the uniform heating or cooling with the localized periodical surface heating. This combination is both important and necessary. As we discussed above, the shortcoming of the localized periodical surface heating is that it may cause the possibility of transition for different frequency disturbances. The present control strategy avoids this shortcoming by adding the uniform heating or cooling during the cancellation of T–S wave. Thus, the remaining wave disturbances will be reduced again and the transition can be further delayed.

Even if the Liepmann control strategy can effectively cancel the T–S wave, it also leads to the risk of instability for other frequency fluctuations. As for our feedback control, this risk of instability can be avoided by adjusting the suitable value of controller gain K_d. The effect of uniform heating is always favorable for stabilizing all fluctuations in the flow field for water. If this effect can surpass the risk of transition caused by the periodical heating (too large K_d), our feedback control will be successfully performed without raising any danger of instability. For the above example, $\bar{q}_w^* = 35.16$ W/m^2 is corresponding to $\epsilon = 0.01$. $\epsilon\hat{q}_w^*$ is approximately proportional to controller gain K_d and for $K_d = 2000$, $\epsilon\hat{q}_w^* = 2264$ W/m^2. Then for the safety of control, we can choose $\epsilon|\hat{q}_w^*| = \bar{q}_w^*$ and it results in $K_d = 31$. Then the input heat fluxes on the two walls are all heating the flow. Thus, the risk of instability for other frequency disturbances are well avoided. This is a very conservative control strategy. In fact, the controller gain K_d can be much larger than 31. It depends on the conditions of the flow which is being controlled.

Fig. 3 shows the dependence of critical wave number α_c, minimum unstable Reynolds number Re_c, control position y_c and control phase angle θ_c on the controller gain K_d with $\epsilon = -0.01$ (cooling water). It is shown that as K_d increases, α_c increases and Re_c and θ_c decreases, and y_c like that in the above heating case is a near constant. It is concluded that this feedback control strategy is also powerful to make laminar flow more unstable and speed up its transition procedure. From Eq. (13), it is found that for the same controller gain K_d, the effect from the controlled temperature disturbance $\hat{T}_0$ will increase as the decrease of $\alpha Re\, Pr$. Large temperature disturbance effect means big value of $\omega_{\mu 2i}$ which is directly related to disturbance growth rate, see the following equation (22). Thus, this feedback control is more powerful for unstabilizing flow than stabilizing it. This large temperature fluctuation effect is also shown in Figs. 4(b), 5(a) and 6(a) where the large $\omega_{\mu 2i}$ appears.

From Eq. (8), $\hat{\omega} = \alpha\hat{c}$ can be expanded as:

$$\omega_i = \omega_{0i} + \epsilon K_\mu(\omega_{\mu 1i} + \omega_{\mu 2i}). \tag{22}$$

Fig. 4 shows how the $\omega_{\mu 1i}$ and $\omega_{\mu 2i}$ depends on α and Re when $K_d = 0$. Due to the advantage of the expansion method, the small heat transfer parameter ϵ is well separated out of our problem (for other problems are the same). This means the $\omega_{\mu 1i}$ and $\omega_{\mu 2i}$ of Figs. 4–6 are independent of ϵ. Fig. 4 also shows that the $\omega_{\mu 2i}$ is about 10% of the $\omega_{\mu 1i}$ for $K_d = 0$. Our feedback control is achieved by improving the effects of $\hat{T}_0$ which causes the dependence of $\omega_{\mu 2i}$ on K_d. The $\omega_{\mu 1i}$ from the mean temperature effects are independent of K_d.

Fig. 5 shows $\omega_{\mu 2i}$ and θ_c depend on α and Re for $K_d = 1000$. It is found that the control position y_c is a near constant for all cases we studied. Thus, the results of Figs. 5 and 6 are computed by choosing the optimal $y_c = 0.87$. It shows that the $\omega_{\mu 2i}$ increases with the decrease α and Re. This is explained as that of Fig. 4(b). θ_c depends linearly on α and Re. It becomes large with decreasing α and increasing Re. As the controller gain K_d is increased, the temperature fluctuation

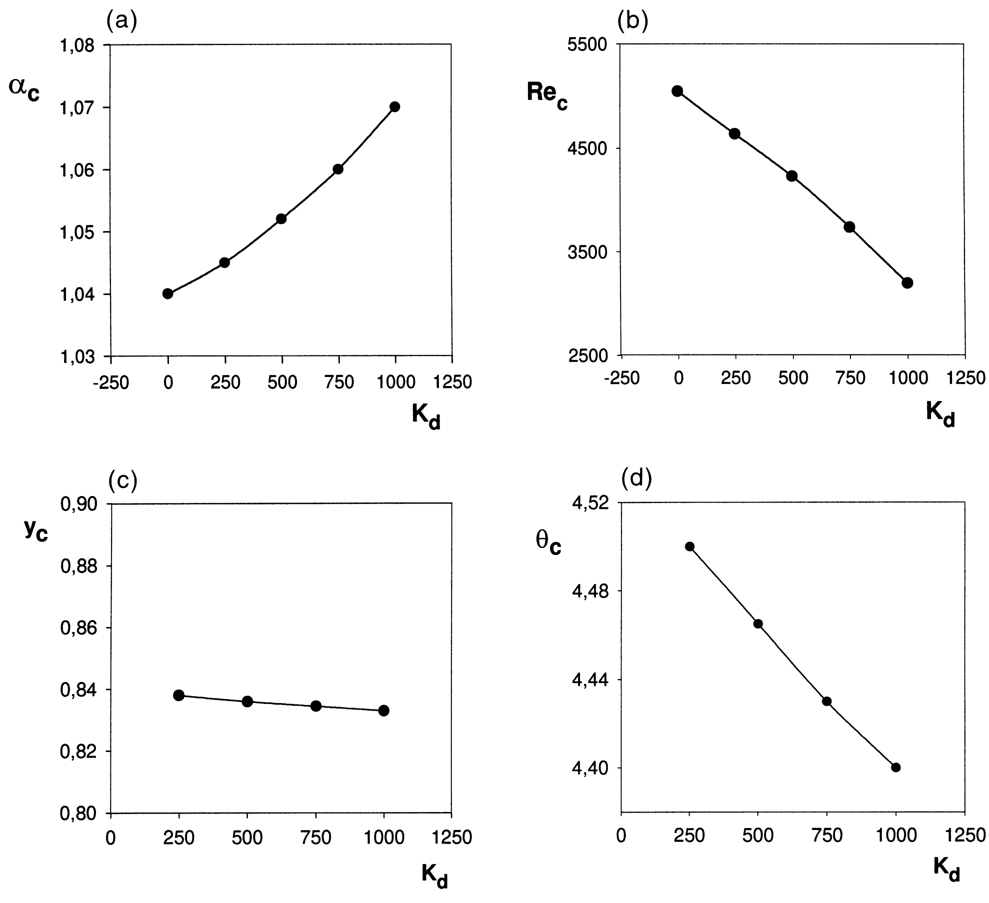

Fig. 3. The relation of critical wave number α_c, minimum unstable Reynolds number Re_c, optimal control position y_c and control phase angle θ_c with controller gain K_d. $\epsilon = -0.01$ and $Pr = 7$. (a) α_c depends on K_d, (b) Re_c depends on K_d, (c) y_c depends on K_d, (d) θ_c depends on K_d.

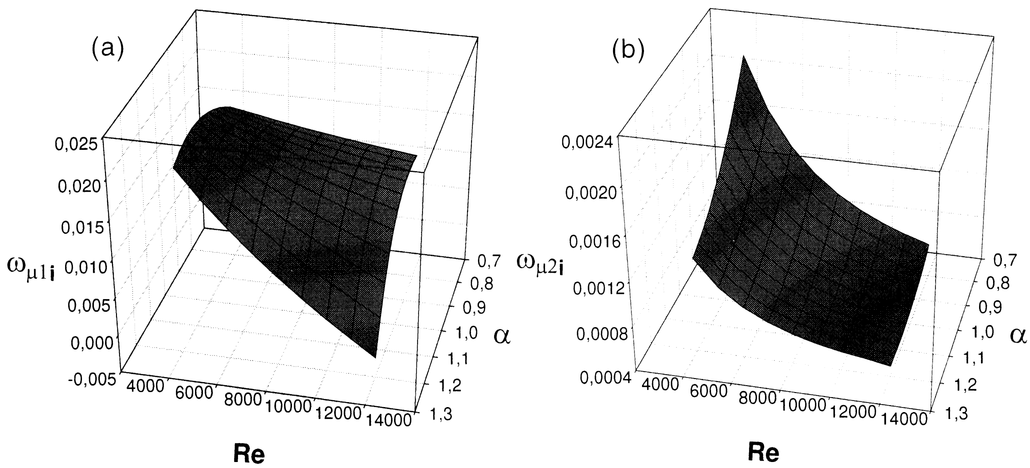

Fig. 4. The relation of coefficients $\omega_{\mu 1i}$ and $\omega_{\mu 2i}$ with Reynolds number Re and wave number α. $K_d = 0$ and $Pr = 7$. (a) $\omega_{\mu 1i}$ depends on Re and α, (b) $\omega_{\mu 2i}$ depends on Re and α.

$\hat{T}_0$ is greatly enhanced and then it causes the increase of $\omega_{\mu 2i}$. Now our goal to control transition is achieved.

Fig. 6 shows $\omega_{\mu 2i}$ and θ_c depend on α and Re for $K_d = 2000$. The results are similar to those of $K_d = 1000$. At this time, the $\omega_{\mu 2i}$ is further increased and the feedback control effect is improved.

7. Conclusions

It is shown that the new feedback control strategy is a very powerful method for laminar flow control. It has an advantage over the method of Liepmann. It can control not only the T–S wave disturbance but also all other fluctuations. Thus, it avoids the shortcoming of Liepmann control and the transition can be further delayed. On the other side, the small perturbation parameter ϵ, which is related to the uniform boundary heat flux, is well separated out from our problem by applying the expansion method. As a result, the heat flux independent coefficients $\omega_{\mu 1i}$ and $\omega_{\mu 2i}$, which reflect the temperature dependent viscosity effect, can be obtained.

In this study, the plane Poiseuille water flow is chosen as example. It is undoubted that this feedback con-

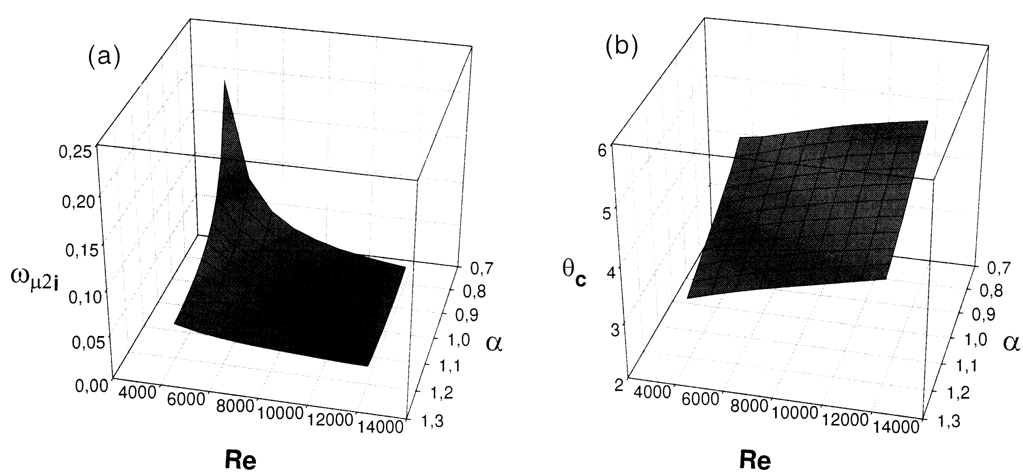

Fig. 5. The relation of coefficient $\omega_{\mu 2i}$ and control phase angle θ_c with Reynolds number Re and wave number α. $K_d = 1000$, $y_c = 0.87$ and $Pr = 7$. (a) $\omega_{\mu 2i}$ depends on Re and α, (b) θ_c depends on Re and α.

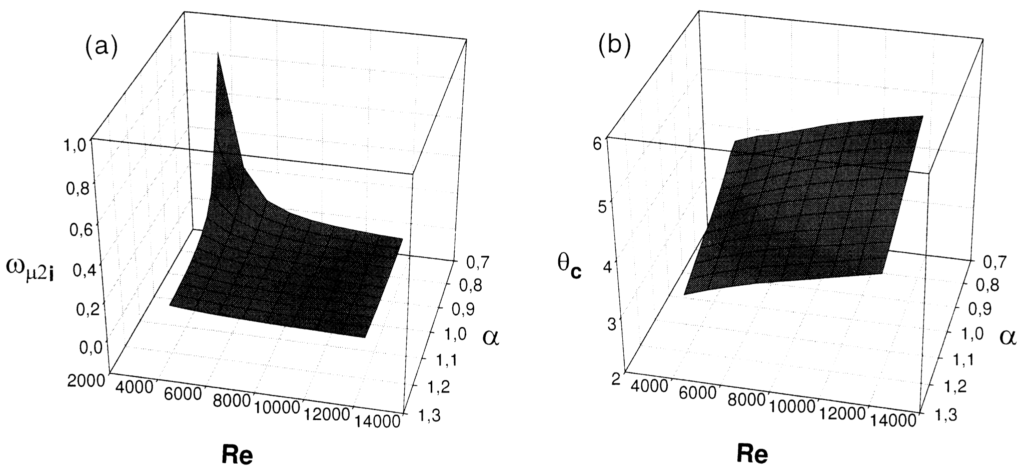

Fig. 6. The relation of coefficient $\omega_{\mu 2i}$ and control phase angle θ_c with Reynolds number Re and wave number α. $K_d = 2000$, $y_c = 0.87$ and $Pr = 7$. (a) $\omega_{\mu 2i}$ depends on Re and α, (b) θ_c depends on Re and α.

trol strategy can also be applied to control other flow fields and fluids.

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